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An Effort Towards The Generalized Hypergeometric Polynomials of Two Variables

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Abstract: In the present paper, we unify the generalized Hypergeometric polynomial Set $S_n(x, y)$ by means of a generating function, which contains Appell function of two variables in the notation of Burchnall and Chaundy [4]. This polynomial Set covers as many as forty-one orthogonal and non-orthgonal polynomials and have been deduced as particular cases.

The newly defined generalized Hypergeometric polynomial Set $S_n(x,y)$may be immense use in new phase of Mathematics relvent to Physics,Chemistry,Engineering and Social sciences .

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1. Introduction

We study and define the generalized hypergeometric polynomial set $S_n(x, y)$ by means of the generating functions,

$$e^{\lambda yt}\; F\left[\begin{matrix}(G_r); & \\ & \lambda_1 y^{e_1} t^{e_1} \\ (H_s); & \end{matrix}\right] \times F\left[\begin{matrix}(a_p);(A_h);(C_u) & \\ & \lambda_3 x^{e_3} t, \lambda_2 x^{e_2} y^{-e_2} t^{e_2} \\ (b_q);(B_k);(D_v) & \end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} S_{n,e_1;e_2;e_3;(H_s);(b_q);(B_k);(D_v)}^{\lambda;\lambda_1;\lambda_2;\lambda_3;(G_r);(a_p);(A_h);(C_u)}(x,y)\,t^n \qquad \ldots (1.1)$$

Where λ, λ_1, λ_2, λ_3 are real and e_1, e_2,e_3 are positive integers.

The left hand side of (1.1) contains Appell function of two variables in the notation of Burchnall and Chaundy [4]. The polynomial set contains a number of parameters, for simplicity, we shall denote.

$$S_{n,e;e_1;e_2;e_3;(H_s)(b_q);(B_k);(D_v)}^{\lambda;\lambda_1;\lambda_2;\lambda_3;(G_r);(a_p);(A_h);(C_u)}(x,y) \text{ by } S_n(x, y).$$

Where n denote the order of the polynomial set.

NOTATIONS:

(i) $(m) = 1, 2, 3, \ldots\ldots, m.$

(ii) $(A_p) = A_1, A_2, A_3, \ldots\ldots A_p.$

(iii) $[(A_p)] = A_1, A_2, A_3 \ldots\ldots A_p.$

(iv) $[(A_p)]_n = (A_1)_n\, (A_2)_n, (A_3)_n \ldots\ldots (A_p)_n.$

(v) $\Delta(a,b) = \frac{b}{a}, \frac{b+1}{a}, \ldots\ldots \frac{b+a-1}{a}.$

(vi) $\Gamma(a \pm b) = \Gamma(a+b)\Gamma(a-b).$

(vii) $\Gamma_*\Gamma_*(a+b) = \Gamma(a+b)\Gamma(a+b).$

(viii) $K = \frac{[(a_p)]_n [(A_h)]_n (\lambda_3 x^{e_3})^n}{[(b_q)]_n [(B_k)]_n \quad n!}$

$$K_{(m)} = \frac{(-n)_m \left(1-(b_q)-n\right)_m \left(1-(B_k)-n\right)_m \lambda^m y^m (-1)^{(b-k+\rho-p)}}{\left(1-(a_p)-n\right)_m \left(1-(A_n)-n\right)_m \quad m!}$$

$$K_{(m_1)} = \frac{\Delta_{m_1}[e_1;-n+m]\Delta_{m_1}\left[e_1;1-(b_q)-n+m\right]}{\Delta_{m_1}\left[e_1;1-(a_p)-n+m\right]\Delta_{m_1}\left[e_1;1-(A_h)-n+m\right]}$$

$$\times \frac{\Delta_{m_1}\left[e_1;1-(B_k)-n+m\right]\left[(G_r)_{m_1} y^{e_1 m_1}\lambda_1^{m_1}\right]}{\left[(H_s)\right]_{m_1} m_1!\left(\lambda_3 x^{e_3}\right)^{e_1 m_1}} \times (-e_1)^{e_1(h-k+q-p+1)m_1}$$

$$K_{(m_2)} = \frac{\Delta_{m_2}[e_2;-n+m+e_1 m_1]\Delta_{m_2}\left[e_2-1;1-(b_q)-n+m+e_1 m_1\right]}{\Delta_{m_2}\left[e_2-1;1-(a_p)-n+m+e_1 m_1\right]}$$

$$\times \frac{\Delta_{m_2}\left[e_2;1-(B_k)-n+m+e_1 m_1\right]\left[(C_u)\right]_{m_2}\left(\lambda_2 x^{e_2}\right)^{m_2}}{\Delta_{m_2}\left[e_2;1-(A_h)-n+m+e_1 m_1\right]\left[(D_v)\right]_{m_2} m_2!}$$

$$\times \frac{\{-(e_2-1)\}^{(e_2-1)(q-p)m_2} (-e_2)^{e_2(k-h+1)m_2}}{\left(\lambda_3 x^{e_3} y\right)^{e_2 m_2}}$$

2. Theorem

If the Hypergeometric polynomial set $S_n(x, y)$ is defined by (1.1) then we can write

$$S_n(x,y)=\sum_{\substack{m,m_1,m_2>0\\ m+e_1m_1+e_2m_2\le 0}}\frac{\Delta(m_1,m_2)}{(n-m-e_1m_1-e_2m_2)!} \quad \text{... (2.1)}$$

Where $\Delta(m_1,m_2)=\dfrac{[(a_p)]_{n-m-e_1m_1-(e_2-1)m_2}}{[(b_q)]_{n-m-e_1m_1-(e_2-1)m_2}}$

$$\times\frac{[(A_h)]_{n-m-e_1m_1-e_2m_2}[(G_r)]_{m_1}[(C_u)]_{m_2}}{[(B_k)]_{n-m-e_1m_1-e_2m_2}[(H_s)]_{m_1}[(D_v)]_{m_2}}$$

$$\times\frac{x^{m_2e_2}\lambda^m\lambda_1^{m_1}\lambda_2^{m_2}\left(\lambda_3x^{e_3}\right)^{n-m-e_1m_1-e_2m_2}\ y^{m+e_1m_1+e_2m_2}}{m!\quad m_1!\quad m_2!\ \ (n-m-e_1m_1-e_2m_2)!}$$

Proof : On using the expansion [4]

$$F\begin{bmatrix}(a_r);(A_p);(\alpha_u) & \\ & (x,y) \\ (b_s);(B_q);(\beta_v) & \end{bmatrix}$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{[(a_r)]_{n+k}[(A_p)]_n[(\alpha_u)]_k\,x^ny^k}{[(b_s)]_{n+k}[(B_q)]_n[(\beta_v)]_k\,n!k!} \quad \text{... (2.2)}$$

Now from (1.1), we arrive at

$$\sum_{n=0}^{\infty}S_n(x,y)t^n=\sum_{n=0}^{\infty}\sum_{m,m_1m_2=0}^{\infty}\frac{(\lambda yt)^m[(G_r)]_{m_1}\left(\lambda_1y^{e_1}t^{e_1}\right)^{m_1}}{[(H_s)]_{m_1}\qquad m_1!}$$

$$\times\frac{[(a_p)]_{n+m_2}[(A_h)]_n[(C_u)]_{m_2}\left(\lambda_3x^{e_3}t\right)^n\left(\lambda_2x^{e_2}y^{-e_2}t^{e_2}\right)^{m_2}}{[(b_q)]_{n+m_2}[(B_k)]_n[(D_v)]_{m_2}\ n!\ \ m_2!}$$

$$=\sum_{n=0}^{\infty}\sum_{m,m_1,m_2=0}^{\infty}\frac{[(a_p)]_{n+m_2}[(A_h)]_n[(C_u)]_{m_2}\lambda^m\ \lambda_1^{m_1}}{[(b_q)]_{n+m_2}[(B_k)]_n[(D_v)]_{m_2}\ m!\ \ m_1!}$$

$$\times \frac{\lambda_2^{m_2} x^{e_2 m_2} y^{m+e_1 m_1} \left(\lambda_3 x^{e_3}\right)^n t^{n+m+e_1 m_1+e_2 m_2}}{y^{e_2 m_2} \quad m_2! \quad n!}$$

$$= \sum_{n=0}^{\infty} \sum_{\substack{m,m_1,m_2=0 \\ m+e_1 m_1+e_2 m_2 \le n}}^{\infty} \frac{\Delta(m_1, m_2)\, t^{n+m+e_1 m_1+e_2 m_2}}{n!} \qquad \ldots (2.3)$$

Now, making use of the Lemma

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} A(k,n) = \sum_{n=0}^{\infty} \sum_{k=0}^{\left[\frac{n}{m}\right]} A(k, n-mk)$$

where *m* is the positive integer. This Lemma is generalized by Rainville [10; P. 57 (37)]

In general

$$\sum_{\substack{m,m_1,m_2 \ge 0 \\ m+e_1 m_1+e_2 m_2 \le n}} \bar{\Delta}(m_1, m_2) \ne \sum_{m_1=0}^{\left[\frac{n}{e_1}\right]} \sum_{m_2=0}^{\left[\frac{n-e_1 m_1}{e_2}\right]} \bar{\Delta}(m_1, m_2)$$

However $\bar{\Delta}(m_1, m_2)$ replaced by $\frac{\bar{\Delta}(m_1, m_2)}{(n-m-e_1 m_1 - e_2 m_2)!}$ we may have

$$\sum_{m,m_1,m_2 \ge 0} \frac{\bar{\Delta}(m_1, m_2)}{(n-m-e_1 m_1 - e_2 m_2)!}$$

$$= \sum_{m=0}^{\infty} \sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]} \sum_{m_2=0}^{\left[\frac{n-e_1 m_1}{e_2}\right]} \frac{\bar{\Delta}(m_1, m_2)}{(n-m-e_1 m_1 - e_2 m_2)!} \qquad \ldots (2.4)$$

Hence, we arrive at

$$\sum_{n=0}^{\infty} S_n(x,y) t^n = \sum_{n=0}^{\infty} \sum_{\substack{m,m_1,m_2 \ge 0 \\ m+e_1 m_1+e_2 m_2 \le n}}^{\infty} \frac{\Delta(m_1, m_2)}{(n-m-e_1 m_1 - e_2 m_2)!} t^n$$

On comparing the co-efficient of t^n from both sides, we get (2.1)

Due to the presence of $(n - m - e_1 m_1 - e_2 m_2)!$ in the denominater, the series (2.1) may be separated by adding some extra terms having values zero as follows:

$$\sum_{\substack{m,m_1,m_2\ge 0\\ m+e_1m_1+e_2m_2\le n}} \frac{\Delta(m_1,m_2)}{(n-m-e_1m_1-e_2m_2)!}$$

$$= \sum_{m=0}^{n}\sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]}\sum_{m_2=0}^{\left[\frac{n-m-e_1m_1}{e_2}\right]} \frac{\Delta(m_1,m_2)}{(n-m-e_1m_1-e_2m_2)!}$$

Hence, we have

$$S_n(x,y) = \sum_{m=0}^{n}\sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]}\sum_{m_2=0}^{\left[\frac{n-m-e_1m_1}{e_2}\right]} \frac{\left[(a_p)\right]_{n-m-e_1m_1-(e_2-1)m_2}}{\left[(b_q)\right]_{n-m-e_1m_1-(e_2-1)m_2}}$$

$$\times\frac{\left[(A_h)\right]_{n-m-e_1m_1-e_2m_2}\left[(G_r)\right]_{m_1}\left[(C_u)\right]_{m_2}\lambda^{m}\ \ \lambda_1^{m_1}}{\left[(B_k)\right]_{n-m-e_1m_1-e_2m_2}\left[(H_s)\right]_{m_1}\left[(D_v)\right]_{m_2}\ m!\ \ m_1!}$$

$$\times\frac{\left(\lambda_2x^{e_2}\right)^{m_2}\left(\lambda_3x^{e_3}\right)^{n-m-e_1m_1-e_2m_2}\ \ y^{m+e_1m_1-e_2m_2}}{m_2!\qquad\qquad (n-m-e_1m_1-e_2m_2)!} \qquad \ldots (2.5)$$

Hypergeometric forms:

Operating the parameters m, m_1 and m_2 we get various hypergeometric forms of (2.5),which are given in the forms of corollaries :

Corollary-I: For $e_2 > 1$, we get

$$S_n(x,y) = K\sum_{m=0}^{n}\sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]} K(m)K(m_1)$$

$$\times F\left[\begin{array}{l}\Delta(e_1;n+m+e_1m_1),\Delta\left(e_2-1;1-(b_q)-n+m+e_1m_1\right),\\ \Delta\left(e_2;1-(B_k)-n+m+e_1m_1\right),C_u;\\ \qquad\qquad \dfrac{\lambda_2x^{e_2}(-e_2)^{e_2(h-k+1)}\left\{-(e_1-1)\right\}^{(e_2-1)(q-p)}}{\left(\lambda_3x^{e_3}y\right)^{e_2}}\\ \qquad \Delta\left(e_2-1;1-(a_p)-n+m+e_1m_1\right),\\ \qquad \Delta\left(e_2;1-(A_h)-n+m+e_1m_1\right),(D_v),\end{array}\right] \qquad \ldots (2.6)$$

Corollary-II : For $e_2 = 1$, we have

$$S_n(x,y) = K\sum_{m=0}^{n}\sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]} K(m)K(m_1)$$

$$\times F\left[\begin{matrix} -n+m+e_1m_1, 1-(B_k)-n+m+e_1m_1, (C_u); \\ \dfrac{\lambda_2 x(-1)^{(h-k+1)}}{\lambda_3 x^{e_3} y} \\ 1-(A_h)-n+m+e_1m_1; (D_v), \end{matrix}\right] \quad \ldots (2.7)$$

Corollary-III: For $e_1 > 1$, we have

$$S_n(x,y) = K \sum_{m=0}^{n} \sum_{m_2=0}^{\left[\frac{n-m-e_1m_1}{e_2}\right]} K(m) K(m_2)$$

$$\times F\left[\begin{matrix} \Delta(e_1;-n+m), \Delta(e_1;1-(b_q)-n+m), \\ \Delta(e_1;1-(B_k)-n+m), (G_r); \\ \dfrac{\lambda_1 y^{e_2}(-e_1)^{e_1(k-h+q-p+1)}}{(\lambda_3 x^{e_3})^{e_1}} \\ \Delta(e_1;1-(a_p)-n+m), \\ \Delta(e_1;1-(A_h)-n+m), (H_s), \end{matrix}\right] \quad \ldots (2.8)$$

Corollary-IV: For $e_1 = 1$, we have

$$S_n(x,y) = K \sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]} \sum_{m_2=0}^{\left[\frac{n-m-e_1m_1}{e_2}\right]} K(m_1) K(m_2)$$

$$\times F\left[\begin{matrix} -n, 1-(b_q)-n, 1-(B_k)-n; \\ \lambda\, y(-1)^{h-k+q-p} \\ 1-(a_p)-n, 1-(A_h)-n; \end{matrix}\right] \quad \ldots (2.9)$$

Particular Cases of (2.6):

On specializing the various parameters involved in hypergeometric forms, a number of known and unknown polynomials can be obtained as particular cases of the set $S_n(x, y)$. Some of them, which are well known, are listed below:

(A) Separating the term corresponding to $\lambda=0$ and putting $r = 0 = s = \lambda_1$ in (2.6), we obtain a number of results on specializing the remaining parameters :

(i) If we set $p = 0 = q = h = k$; $u = 1 = v = e_3$; $y = x$; $e_2 = m$; $\lambda_2 = \mu$; $\lambda_3 = v$; $C_1 = a_r$, $D_1 = b_s$, we have

$$(C)_n S_n(x,x) = \frac{(vx)^n}{n!} x^n \; {}_{m+r}F_s\left[\begin{matrix} \Delta(m;-n), (a_r); \\ \mu\left(\dfrac{-m}{vx}\right)^m \\ (b_s); \end{matrix}\right]$$

$$= A_n(x) \quad \ldots (2.10)$$

Where $A_n(x)$ are the generalized by Panda [9].

(ii) On making the substitution $p = 0 = q = h = k = u = v$; $e_3 = 1 = \lambda_3$; $\lambda_2 = h$, $y = x$, we have

$$J_n(x,x) = \frac{x^n}{n!}\ {}_mF_0\left[\begin{matrix}\Delta(m;-n); \\ \quad \\ \text{—};\end{matrix}\quad h\left(\frac{-m}{x}\right)^m\right]$$

$$= \frac{1}{n!} g_n^m(x,h) \qquad \ldots (2.11)$$

Where $g_n^m(x,h)$ are the generalized polynomials defined by Gould-Hopper [5].

(iii) On setting $p = 0 = q = h = k = u = v$; $\lambda_2 = -1$, $e_2 = p$; $e_3 = 1 = \lambda_3$; $y = x$ and $x = py$, we arrive at

$$S_n(py,x) = \frac{(py)^n}{n!}\ {}_pF_0\left[\begin{matrix}\Delta(p;-n); \\ \quad \\ \text{—};\end{matrix}\quad -\left(\frac{-1}{y}\right)^p\right]$$

$$= \frac{1}{n!} g_n^p(y) \qquad \ldots (2.12)$$

Where $g_n^p(y)$ are the Bragg polynomials [3].

(iv) On making the substitution $p = 0 = q = h = k = u = v$; $y = x$; $\lambda_3 = 1 = e_3 = e_1$; $e_2 = 2$, $\lambda_2 = -1$; $x = 3y$ we achieve

$$S_n(3y,x) = \frac{(3y)^n}{n!}\ {}_3F_0\left[\begin{matrix}-\frac{n}{3};\frac{-n+1}{3},\frac{-n+2}{3}; \\ \quad \\ \text{—};\end{matrix}\quad \frac{1}{y^3}\right]$$

$$= \frac{1}{n!} h_n^*(y) \qquad \ldots (2.13)$$

where $h_n^*(y)$ are the Humbert polynomials [7].

(v) On taking $p = 0 = q = h = k$; $u = 1 = v = e_3$; $\lambda_3 = 2 = e_2$; $\lambda_2 = -1$, $C_1 = 1 + \alpha$, $D_1 = 1 + \beta$, $y = x$, we find

$$S_n(x,y) = \frac{(2x)^n}{n!}\ {}_3F_1\left[\begin{matrix}-\frac{n}{2};-\frac{n}{2}+\frac{1}{2},1+\alpha; \\ \quad \\ 1+\beta;\end{matrix}\quad -\frac{1}{x^2}\right]$$

$$= \frac{1}{n!} g_n(x) \qquad \ldots (2.14)$$

Where $g_n(x)$ [10; P. 237 (26)] degenerates into the Hermite polynomials for $\beta = \alpha$.

(vi) On taking $p = 0 = q = h = k$, $r = h$; $u = 1 = v = e_3$; $\lambda_3 = 2 = e_2$; $\lambda_2 = -1$, $\alpha_1 = 1 + \alpha$, $\beta_1 = 1 + \beta$, $y = x$, we get

$$S_n(x,y) = \frac{(2x)^n}{n!}\ {}_3F_1\left[\begin{matrix} -\frac{n}{2};-\frac{n}{2}+\frac{1}{2},1+\alpha; \\ \qquad\qquad -\frac{1}{x^2} \\ 1+\beta; \end{matrix}\right]$$

$$= \frac{1}{n!} g_n(x) \qquad \ldots (2.15)$$

Where $g_n(x)$ [10; P. 237 (26)] degenerates into the Hermite polynomials for $\beta = \alpha$.

(vii) If we set $p = 0 = q = h = k = r = s$; $e_3 = 1 = x = y = e_1$; $\lambda_3 = -p$; $e_2 = p$; $\lambda_2 = x$, we achieve

$$S_n(1,1) = \frac{(-p)^n}{n!}\ {}_{p+u}F_v\left[\begin{matrix} \Delta(p;-n),\alpha_1,\alpha_2,\alpha_3\ldots\ldots\alpha_u; \\ \qquad\qquad x \\ \beta_1,\beta_2,\beta_3\ldots\ldots\beta_v; \end{matrix}\right]$$

$$= \frac{(-p)^n}{n!} B_n^p(x), \qquad \ldots (2.16)$$

Where $B_n^p(x)$ are the generalization of the Hermite polynomials by Brafman [2].

(viii) On making the substitution $p = 0 = q = h = k = r = s$; $e_2 = m, e_3 = 1 = e_1 = e$; $\lambda_3 = \delta$; $\lambda_2 = \lambda$ and replacing x and y by $\frac{1}{x^{\frac{c}{\delta}}}$; we arrive at

$$S_n\left(\frac{1}{x^{\frac{c}{\sigma}}},\frac{1}{x^{\frac{c}{\delta}}}\right) = \frac{(-\delta)^n x^{\frac{-nc}{\sigma}} x^{(\delta-1)n}}{n!}\ {}_{u+\delta}F_v\left[\begin{matrix} \Delta(\delta;-n),\alpha_u; \\ \qquad \lambda x^c \\ (\beta_v); \end{matrix}\right]$$

$$= \frac{(-\delta)^n x^{\frac{-nc}{\delta}}}{n!} F_n(x), \qquad \ldots (2.17)$$

where $F_n(x)$ are the polynomials defined by Shah [12].

(ix) If we take $e_2 = m$, $e_1 = 1 = e_3$; $\lambda_3 = \nu$; $\lambda_2 = \mu$, $p = 0 = q = r = s$; $B_k = \beta_q$, $A_h = \alpha_p$, $C_u = a_r$, $= D_v = b_s$ we have

$$S_n(x,y) = \frac{[(\alpha_p)]_m}{[(\beta_q)]_n}\frac{(\nu x)^n}{n!}\ {}_{mq+m+r}F_{mp+s}\left[\begin{matrix} \Delta(m;-n),\Delta(m;1-(\beta_q)-n),(a_r), \\ \qquad \frac{\mu(-m)^{m(q-p+1)}}{(\nu xy)^m} \\ \Delta(m;1-(\alpha_p)-n),(b_s); \end{matrix}\right]$$

$$= B_n(x,y) \qquad \ldots (2.18)$$

where $B_n(x,y)$ are the polynomials defined by Khanna [8].

(B) Separating the term corresponding to $m_1 = 0$ and putting $r = 0 = s = \lambda_1$ in (2.7), we obtain a number of results on specializing the remaining parameters.

(x) On Putting $h = 0 = k = u$; $v = 1 = n = \lambda_3$; $\lambda_2 = -1$, $D_1 = 1+\alpha$ and $\frac{1}{y}$ for y, we achieve

$$S_n\left(1,\frac{1}{y}\right) = \frac{(1+\alpha)_n}{n!}\ {}_1F_1\left[\begin{matrix}-n; \\ & y \\ 1+\alpha; \end{matrix}\right] = L_n^{(\alpha)}(y) \qquad \ldots (2.19)$$

Where $L_n^{(\alpha)}(y)$ are the generalized Laguerre polynomials.

(xi) (a) On taking $h = 0 = u$; $k = 1 = v = e_3$; $\lambda_2 = \frac{1}{2} = \lambda_3$ and $x = \frac{x-1}{x+1} = y$, $B_1 = 1+\alpha$, $D_1 = 1+\beta$ in (2.7), we get

$$(1+\alpha)_n (1+\beta)_n S_n(x,x) = \frac{(1+x)^{-1}(1+\beta)_n}{n!}\left(\frac{x-1}{2}\right)^n$$

$$\times\ {}_2F_1\left[\begin{matrix}-n,-\alpha-n; \\ & \frac{x+1}{x-1} \\ 1+\beta; \end{matrix}\right] = (x+1)^{-n} P_n^{(\alpha,\beta)}(x) \qquad \ldots (2.20)$$

Where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi polynomials.

(b) On putting $h = 0 = u$; $k = 1 = v = e_3$; $\lambda_2 = \frac{1}{2} = \lambda_3 B_1 = 1+\beta$; $D_1 = 1+\alpha$ and $\frac{x+1}{x-1}$ instead of x and y, we get

$$(1+\alpha)_n (1+\beta)_n S_n\left(\frac{x+1}{x-1},\frac{x+1}{x-1}\right)$$

$$= (x-1)^{-n}\left(\frac{x+1}{2}\right)^n \frac{(1+\alpha)_n}{n!}\ {}_2F_1\left[\begin{matrix}-n,-\beta-n; \\ & \frac{x-1}{x+1} \\ 1+\alpha; \end{matrix}\right]$$

$$= (x-1)^{-n} P_n^{(\alpha,\beta)}(x) \qquad \ldots (2.21)$$

Where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi polynomials.

(xii) On taking $h = 0 = k$; $u = 1 = v = e_3 = \lambda_3 = x$; $\lambda_2 = -1$, $D_1 = 1$, $C_1 = -x$ and $\frac{1}{1-e^{-\lambda}}$ for y and then using [5; vol-I, P. 105], we get

$$n!S_n\left(1,\frac{1}{1-e^{-\lambda}}\right) = e^{-n\lambda}\ {}_2F_1\left[\begin{matrix}-n,-x; \\ & 1-e^{\lambda} \\ 1; \end{matrix}\right] = \phi_n(x;\lambda) \qquad \ldots (2.22)$$

where $\phi_n(x; \lambda)$ are the Gottliab polynomials.

(xiii) On taking $h = 0 = k$; $\lambda_3 = 1 = e_3 = x$; $y = \frac{1}{y}$ and replace C_u by (a_p) and (D_v) by (b_q), we get

$$n!S_n\left(1,\frac{1}{y}\right) = {}_{p+1}F_q\left[\begin{matrix} -n,(a_p); \\ (b_q); \end{matrix}\; y\right] = {}_1F_1(-n,b;y) \quad \ldots (2.23)$$

where ${}_1F_1(-n,b;y)$ are the Abdul-Halim and Al-Salam [1] polynomials.

(xiv) On making the substitutions $h = 0 = k$; $u = 1 = v = e_3 = x = \lambda_3$; $\lambda_2 = 1-c$, $y = c$; $C_1 = -x$, $D_1 = \beta$, we arrive at

$$(\beta)_n S_n(1,c) = \frac{(\beta)_n}{n!}\,{}_2F_1\left[\begin{matrix} -n,-x; \\ \beta; \end{matrix}\; 1-\frac{1}{c}\right]$$

$$= \frac{1}{n!}m_n(\alpha;\beta,c) \quad \ldots (2.24)$$

where $m_n(\alpha, \beta, c)$ are the Meixner polynomials.

(xv) On making the substitutions $h = 0 = k = u$; $e_3 = 1 = \lambda_3 = x$; $v = 2$; $D_1 = 1 + \alpha$, $D_2 = 1 + \beta$; $\lambda_2 = -1$ and $\frac{1}{x}$ for y,

we get

$$S_n\left(1,\frac{1}{x}\right) = \frac{1}{n!}\,{}_1F_2\left[\begin{matrix} -n; \\ 1+\alpha,1+\beta \end{matrix}\; x\right] = \frac{1}{n!}f_n(x) \quad \ldots (2.25)$$

where $f_n(x)$ is related to Bateman function[10; P. 243 (4)].

(xvi) If we set $h = 0 = k$; $u = 1 = e_3 = \lambda_3 = x$, $\lambda_2 = -1$; $v = 2$, $C_1 = 1 + \beta$, $D_1 = 1$, $D_2 = 1 + \alpha$, and $\frac{1}{x}$ for y, we get

$$S_n\left(1,\frac{1}{x}\right) = \frac{1}{n!}\,{}_2F_2\left[\begin{matrix} -n,1+\beta; \\ 1,1+\alpha; \end{matrix}\; x\right] = \frac{1}{n!}\phi_n(x) \quad \ldots (2.26)$$

where $\phi_n(x)$ [10; P. 235 (12)] reduces to simple Laguerre polynomials for $\alpha = \beta$.

(xvii) On setting $k = 0 = u = v$; $h = 1 = e_3 = x_3 = \lambda_2$; $\lambda_3 = -1$; $A_1 = 1 + \lambda$, and $\frac{1}{y}$ for y we get

$$S_n\left(1,\frac{1}{y}\right) = \frac{(1+\lambda)_n(-1)^n}{n!}\,{}_1F_1\left[\begin{matrix} -n; \\ -\lambda-n; \end{matrix}\; -y\right] = A_n^{(\lambda)}(y) \quad \ldots (2.27)$$

where $A_n^{(\lambda)}(y)$ are the Srivastava's polynomials [11].

(xviii) If we take $k = 0 = u = v$; $p = 1 = e_3 = x = \lambda_2 = \lambda_3$; $A_1 = -\lambda$, and $\frac{1}{x}$ for y, we get

$$S_n\left(1, \frac{1}{x}\right) = \frac{(-\lambda)_n}{n!}\ {}_1F_1\left[\begin{matrix} -n; \\ & x \\ 1+\lambda-n; \end{matrix}\right] = f_n(x) \quad \ldots (2.28)$$

where $f_n(x)$ [10; P. 245 (12)] are the Pseudo-Laguerre polynomials.

(xix) On making the substitutions $h = 0 = u$; $k = 1 = v = e_3 = y$; $B_1 = \lambda + \frac{1}{2} = D_1$; $\lambda_2 = \frac{1}{2} = \lambda_3$ and writing $\frac{x+1}{x-1}$ for x and y, we get

$$\left(\lambda + \frac{1}{2}\right)_n (2\lambda)_n\, S_n\left(\frac{x+1}{x-1}, \frac{x+1}{x-1}\right)$$

$$= \frac{(x-1)^{-n}(2\lambda)_n}{n!}\left(\frac{x+1}{2}\right)^n {}_2F_1\left[\begin{matrix} -n, -\frac{1}{2}-\lambda-n; \\ & \frac{x-1}{x+1} \\ \lambda+\frac{1}{2}; \end{matrix}\right]$$

$$= (x-1)^{-n} C_n^{\lambda}(x) \quad \ldots (2.29)$$

Where $C_n^{\lambda}(x)$ are the Gegenbauer polynomials.

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